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SINGULARITIES AND EXOTIC SPHERES[1]

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In: Conference Report 27/1996, Singularities 14.07.-20.07.1996, Mathematical Research Institute Oberwolfach, "Brieskorn–Day". 16.07.1996, Lecture on the occasion of the 60th birthday of Egbert Brieskorn, short version.

Report on the academic year 1965/66. Brieskorn is C.L.E. Moore Instructor at M.I.T., Jänich is at Cornell University, then at IAS in Princeton. I am in Bonn. There is an extensive correspondence. From 30.09.–07.10.1965 I'm at a conference in Rome (report on Brieskorn's simultaneous resolutions). Brieskorn's letter from 28.09.1965 reaches me there: *"I have made the somewhat confusing discovery in recent days that there may be 3 - dimensional normal singularities that are topologically trivial. I discussed the example with Mumford this afternoon, and he had not found a mistake by this evening: here it is:* $X = \{x \in \mathbf{C}^4 \mid x_1^2 + x_2^2 + x_3^2 + x_4^3 = 0\}$*.".* Proof by resolution and calculation of all invariants of the neighbourhood boundary. In the Proc. Nat. Aca. Sci. USA appears the more general example $x_1^2 + \cdots + x_k^2 + x_{k+1}^3 = 0$ (k odd).

Report on the extensive correspondence that follows, about Brieskorn's discovery of the work of Pham, which allows him to prove Milnor's assertion in a letter to Nash - Milnor to Nash on 13.04.1966: *"The Brieskorn example is fascinating. After starting at if for a while, I think I know which manifolds of this type are spheres but the statement is complicated and a proof does not exist. Let* $\sum(p_1, \ldots, p_n)$ *be the locus* $z_1^{p_1} + \cdots + z_n^{p_n} = 0, |z_1|^2 + \cdots + |z_n|^2 = 1$ *where* $p_j \geq 2$ *..."* Then Milnor gives the condition a) or b) for the exponents. – Gradually it becomes clear to all parties that for the determination of the differentiable structure the calculation of the signature of $z_1^{p_1} + \cdots + z_n^{p_n} = 1 (n \geq 3,\, n \text{ odd})$ is required. There are several letters from Brieskorn to me and vice versa. Brieskorn writes his paper for the Inventiones Vol. 2 (1966). In this context, he also studied $(2, 3, 5, 30), 30 =$ Coxeter number of E_8, and he finally accomplished the small resolutions of this singularity in curves according to the E_8–tree and thus the simultaneous resolution of the surface families $x_1^2 + x_2^3 + x_3^5 + t^{30} = 0$ (parameter t) and the remaining case of his paper in Math. Ann. of 1966 (about which I reported in Rome). Understanding was achieved within the framework of the root systems and the Weyl group (Brieskorn's letter to Mrs. Tjurina dated 13.09.1966) – Jänich had studied $O(n)$–manifolds $W^{2n-1}(d)$ (two orbit types with isotropy groups $O(n-2), O(n-1)$ and orbit space D^2, S^i), and classified them as well as the knot manifolds $M^{2n+1}(k)$ on which $O(n)$ operates (three orbit types $O(n-2), O(n-1), O(n)$ with orbit space $D^4, S^3 - k, k$ (k the knot)). I bring the two located in the USA together by a report from March 1966, e.g. $W^{2n-1}(d)$ is $\sum(2, \ldots, 2, d)$ and M^{2n+1} (torus knot 3, 5) is $\sum(2, \ldots, 2, 3, 5)$. Brieskorn writes on 29.03.1966: *"Klaus Jänich and I had not noticed anything about this connection of our work, and I was completely overjoyed, how you brought things together."*

I had the same joy here in Oberwolfach, to be able to tell about it.

[1]Translated from the german article in: Tagungsbericht 27/1996, Singularitäten 14.07.-20.07.1996, Mathematisches Forschungsinstitut Oberwolfach.