Some of Infinity

COMING SOON

Peak[illegible]n the Landscape of Mat[illegible]matics

David Craft

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Acknowledgments

This book came about from trying to convince various friends that math is fun and cool. I realize saying "math is cool" is probably not cool. But saying math is fun, or math is interesting, or math is neat, well, those don't sound all that cool either. Anyhow, I always do my best to show people what it is I like about math, and finally I decided to write a book about that. Georgia Kennedy was the person who finally got me to start this project. She's an artist and as many good artists, she's interested in all things, which for me fortunately includes math. Early scribbles on napkins at River Gods led to this book.

Many friends and family members read drafts along the way and helped tighten it. My mom Paula Craft and my brother Jay Craft both slogged through it more than once. My mom claimed most of it was over her head, but in the weeks after reading it she would mention some things about probability or the Rubik's cube, so I know

some things got through. My brother has seen me try to explain the Monte Hall problem or $0.\overline{9999}$ type of things to people, so he was a ready and willing reader with lots of great suggestions. Pete Galea, my college roommate who is currently showing people fishes off the coast of Thailand, was a key early teacher of mine, helping me get a grip on many concepts that I first became exposed to in high school but didn't really understand until Pete helped me along. Greg Aloe and Paul Lang filled early drafts with helpful red pen markups. Thanks to Eduardo Espada for drawing Probability Man. Some more friends went for a read but life got in the way, or they decided they actually hated math, but I'm grateful for their feedback as well. I thought I could hook just about everyone but I've since changed my tune on that one.

I'd like to thank a few more people along my journey that made me think about math in new ways. In high school, I was fortunate to be there when Matt Plummer first arrived. He was an enthusiastic young teacher who gave me extra problems to push me along in geometry and trigonometry. I still have vague memories of one complicated problem involving the geometry of a baseball field. This was a first example for me of having just the right amount of information to figure out a problem. Bob Condon was another geometry teacher of mine. He started the class with "What is a point?", which was an early example of the importance of clear definitions in mathematics, and the odd

difficulty of defining "obvious" concepts. In college, things ramped up fast. Typical grades in the calculus courses taught by Eva Kallien where 40 or 50 out of 100. I got used to getting such grades, along with everyone else, and got very excited about getting a 60. This taught me that, for everything we were learning in those classes, there was still a ton more to go. We were only peering at some of infinity. Paul Dupuis, another math teacher at Brown, blew my mind when he took us through a proof that something existed, without actually producing that something. This is called an existence proof. I've never had to do one since, but I still remember being stirred by that idea. Allan Bower, my thesis advisor at Brown, taught me independence, a key skill for researchers to gain early. He guided me with a strong mind, teaching me the ropes of writing software to compute the behavior of metals subjected to external pressures. His trust in me and guidance was formative.

Some great teachers that come to mind from graduate school are Dimitris Bertsimas, Dimitri Bertsekas, Rob Freund, John Tsitsiklis, John Wyatt, and Asuman Ozdaglar. I can still hear Bertsimas say "attack it" meaning, once you set up a problem enough to know what it is you are trying to solve, it's time to dive in and do it. These teachers and their fearlessness regarding math, sometimes assigning homework whose solutions required pages of equations for one problem, stays with me. Thanks to all of you for defying the oft heard notion that university professors don't

care about teaching, only research. You all showed me that this is not true. Thanks to Ayla Ergun and Kevin Amonlirdviman for making the sweating out of solving those problem sets a lot more fun.

Most recently along my mathematical journey is my boss and colleague Thomas Bortfeld, a physicist by training and also a fine mathematician. He continues to give me freedom to explore any idea that comes to mind, both at work and outside.

David Craft
Cambridge, MA
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Chapter 1

Introduction

Math to me is not quickly computing the tip on a $56 dinner or figuring out if one loan is better than another. Those things I consider arithmetic. Mathematicians are notoriously bad at arithmetic because mathematics is about charting new ground in the sea of mathematical logic and knowledge. To chart new ground means to understand the old ground first and move on in search of new mysteries. Most mathematicians are pretty awful at small time arithmetic ("What's 114 minus 87?") because once they understand how one would figure out such a thing in general, they move on without bothering to master it.

To many people math is thought of as a dull field and I think this is because grade school math is often quite boring. And then, a family vacation or a flu comes along

that takes you out of school for a week and suddenly, not only is it dull, but you are lost and starting to get bad grades. In no time "I hate math" is ringing through the hallways, and most people–once they escape the clutches of their math requirements–never turn back to look at it again.

This is unfortunate because math, as you will see, rather than being dull, is full of strange surprises and beauty and demonstrates astounding human reasoning. Math is a discipline which can make us smarter. I like to hope–and I admit that this might be wishful thinking–that as the human race evolves it gets, on the whole, smarter and we will eventually rectify the numerous problems that we have created: the massive inequalities across the globe and across the cities regarding wealth and opportunities, the daily widespread destruction of life on earth due to manipulation of habitats and the environment for our near-term convenience. We've come far. We have built impressive things. But we have a long way to go regarding how we treat each other and the plants and animals around us. We are smart and we are problem solvers, but we usually apply our intelligence where there is money to support the work.

What does all this have to do with math? In my optimistic hours, I like to believe that logic is a key component to both ethics and to social policy making, which are critical components to making the human race a more respectable lot. One can argue that math by itself will solve none of these. But, I think that to learn to be logical and

to examine all the ifs, ands, and buts that go into designing complicated systems and making large scale decisions, practice is required, and math is a fantastic way to exercise the brain.

I also accept the position made famous by the 20th century English mathematician G.H. Hardy: math for the sake of math. Hardy saw math similar to other endeavors such as music and art which should be pursued due to their inherent beauty and interest. Of course, it would be great if we humans could fix the major human failings, the ones that provide content for our newspapers every day (and the frustrating part is that we can solve them– we have all the technologies we need–but the social and political institutions we have built up prevent and even reverse progress in this arena), but I don't think that we should put on hold all other endeavors like art, music, math, literature, and sports until we do so. We'd be waiting a long time.

Math is the purest form of human reason. But strict adherence to reason alone wouldn't suffice to make math interesting. What is interesting is that starting with only a few primitive accepted ideas, such as numbers, and a few rules, such as how to add them together, logic, plus creativity and pulling together ideas from differing disciplines, can lead us to mind bending truths and ideas.